

**Exam II**

Name: \_\_\_\_\_

Calculators not allowed. Answers do not have to be completely simplified, especially if doing so requires a calculator(duh). Read **all** of the questions carefully before you begin. Do the easiest ones first. Then return to the rest. If you get stuck on a problem, leave it and come back when everything else is finished. Then re-read it, draw a picture, etc. until it makes sense.

**1.[25 pts total]** A remote village in the Himalayas was first settled in 1500 A.D. by 200 farmers. In one year the population grew to 220.

**(a)[4 pts]** Assume that the population grew exponentially. Write down a population function of the form  $P(t) = P_0a^t$  for the village.

**(b)[4 pts]** when did the population reach 1000?

**(c)[4 pts]** Calculate the population in the year 1600.

**(d)[4 pts]** Forget parts (a) through (c). Now assume that the population grew quadratically, and write down a population equation of the form  $P(t) = P_0 + at^2$ .

(e)[5 pts] Compute  $P'(t)$  using your favorite method.

(f)[4 pts] Compute  $P'(5)$  and explain, in complete sentences in English, what your solution means in the context of the problem.

**2.[20 pts total]** A ball is thrown from the ground, straight upward at time  $t = 0$ , with an initial velocity of 128 feet per second. As seen in class, the height of the ball as a function of time is given by

$$f(t) = -16t^2 + 128t$$

(a)[5 pts] Graph  $f(t)$  as neatly as you can. Label your axes appropriately. Label all points of intersection of the function with the axes.

(b)[5 pts] When will the ball reach its maximum height? What will this height be?

(c)[5 pts] Calculate  $\frac{d}{dt}f(t)$  and explain its meaning in this problem, as clearly as you can.

(d)[5 pts] When will the ball hit the ground? What will its velocity be right as it hits the ground?(no, not zero)

3.[20 pts total] Compute the following limits:

(a)[6 pts]  $\lim_{x \rightarrow 1} (3x^3 + 5x^2 - 8) =$

(b)[7 pts]  $\lim_{x \rightarrow 3} \left( \frac{x^2 - 9}{x - 3} \right) =$

(c)[7 pts]  $\lim_{x \rightarrow \infty} \left( \frac{x - 1}{x^2 - 1} \right) =$

4.[25 pts] Compute the derivatives of the following functions:

(a)[6 pts]  $f(t) = 1$

(b)[6 pts]  $g(x) = x^2$

(c)[7 pts]  $h(z) = 3z^5 - 2z^2 + 5z - 1$

(d)[6 pts]  $f(x) = \frac{1}{x}$

5.[10 pts] Use the definition of the derivative to show that your answer in 4(b) is correct.

6.[5 pts extra credit] Assume that  $\lim_{h \rightarrow 0} \left( \frac{e^h - 1}{h} \right) = 1$ . Compute the derivative of  $f(x) = e^x$ .