Math 112Q Spring 2004 Section 8
Instructor: Gorjan Alagic

Exam III

Name: ________________________________

Calculators not allowed. Answers do not have to be completely simplified. Read **all** of the questions carefully before you begin. Do the easiest ones first. Then return to the rest. If you get stuck on a problem, leave it and come back when everything else is finished. Then re-read it, draw a picture, work through it slowly step by step, etc.

The more work you show, the better your chances for partial credit. Make sure you answer **all** of the questions posed in each part.

1.[25 pts total] We return to the remote village in the Himalayas, first settled by 200 farmers. Historical data tells us that the population behaves according to $P(t) = 200 + 20t^2$, where $t = 0$ in the year 1500.

(a)[5 pts] What is the population in 1510? Is the population increasing or decreasing at that point in time? At what rate is it changing?

(b)[5 pts] Is the **rate of change** in 1510 increasing or decreasing? Explain your answer clearly.

(c)[5 pts] Graph the population as a function of time. Mark your axes appropriately. Mark all intersections of the graph with the axes. Make sure you graph what happens for negative t as well.

(d)[**5 pts**] On what intervals is $P(t)$ increasing? Decreasing? Concave up? Concave down?

(e)[**5 pts**] Compute $P''(15)$ and explain, in complete sentences in English, what your solution means in the context of the problem. Think carefully about what the second derivative means.

2.[**25 pts total**] Spaceprobes launched from Earth often have to follow very complicated trajectories, in order to avoid (or take advantage of) the gravitational pulls of surrounding planets. Suppose that a probe is launched in 2010, and that NASA has calculated that its distance from Earth will behave according to the following function:

$$f(t) = t^3 - 8t^2 + 15t$$

where t is in years and $t = 0$ in 2010. You may assume that when $f(t)$ is negative, the probe has passed the Earth in the opposite direction of its launch.

(a)[**5 pts**] Factor $f(t)$ completely. In what years will the probe pass the Earth?

(b)[**5 pts**] Graph the distance of the probe from Earth as a function of time. Label the axes appropriately. Mark all intersections of the graph with the axes.

(c)[5 pts] Use the product rule to differentiate $f(t)$ using its factored form from part (a). Then differentiate it in the expanded version and compare your answers.

(d)[5 pts] Compute $f''(t)$. Describe clearly the meaning of the second derivative in this problem.

(e)[5 pts] On what intervals is $f(t)$ concave up? Concave down? Calculate the time at which its concavity changes. What is the practical meaning of this point?

3.[30 pts] Compute the derivative of the following functions:

(a)[6 pts] $f(t) = sin(t^2)$

(b)[6 pts] $f(\theta) = 5^{sin(\theta)}$

(c)[6 pts] $g(x) = xe^{x^3}$

(d)[6 pts] $h(z) = 2cos^2(z) + ln(e^2)sin^2(z)$

(e)[6 pts] $f(x) = \frac{ln(cos(x))}{tan(x)}$

4.[10 pts] Compute the second derivative of the following functions:

(a)[5 pts] $f(x) = cos(x)sin(x)$

(b)[5 pts] $g(z) = e^{-x^4}$

5.[10 pts] Prove that the derivative of $ln(x)$ is $\frac{1}{x}$. Start by setting $y = ln(x)$ and solving for x.

6.[5 pts extra credit] Prove that the derivative of $arcsin(x)$ is $\frac{1}{\sqrt{1-x^2}}$.